Lecture 8

7.1: Integration by Parts

In Calculus I, you reversed the chain rule:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\,g'(x)$$

and got u-substitution:

$$\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$$

$(u = g(x))$.

This semester, we reverse the other key differentiation rule: the product rule:

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

Integrating this gives

A slight rearrangement of terms gives:

Integration by Parts

Incorporating limits of integration, we have

This is THE BIGGEST GUN you have to attack integration problems. In fact, this tool is so important, it is heavily used in current research in analysis and (partial) differential equations! Before getting into tricks, let's do a simple example:

Ex: $\int_1^4 xe^{2x}\,dx$

Sometimes, it may be necessary to apply IBP two (or more!) times:

Ex: $\int x^2 \cos x\,dx$

Another common trick is letting $dv = dx$:

Ex: ⓐ $\int \arctan(x)\,dx$

ⓑ $\int \ln(x)\,dx$

Finally, another common trick involves the recurrance of an integral:

Ex: $\int e^x \sin x\,dx$

Since IBP is your most powerful tool, you should use it only when other methods won't work

1) Is there a formula for the integral?

e.g. $\int x^n dx$, $\int \cos x \, dx$, $\int e^{ax} dx$, etc. ...

2) Can the integral be done with a u-substitution?

e.g. $\int x e^{x^2} dx$, $\int \frac{\ln x}{x} dx$, $\int \frac{3x}{1+x^2} dx$, etc. ...

3) If you do use IBP, there is an acronym that can help you choose u & dv. Let u be whatever shows up first, and dv be the rest.

L I A T E

- **L**ogarithm
- **I**nverse Trig.
- **A**lgebraic
- **T**rigonometric
- **E**xponential

Integration by parts can also be used to obtain the following reduction formulas:

$$\int \sin^m x \, dx$$

$$\int \cos^m x \, dx$$